ALM 2017

Continue with the data from Exercise 1. Today is still 1st January 2013.

3. Stochastic term structure model

I have fitted a CIR model to the yield curve.There are a number of new worksheets in yellow:

CIR_model shows the model and compares with the "empirical".

CIR_chart is a graph.

CIR_Immunisation is immunisation using the CIR model.

CIR_simulation is 1000 simulations of the yield rate and present values at a given time in the future.

CIR_VaR calculates VaR and TailVaR of liabilities, assets and surplus in the future.

CIR_chart_percentiles charts the percentiles of present values of liabilities, assets and surplus.

There are three exercises:

3.1 Understand what is going on in the worksheets.

3.2 Use different immunising portfolios and see how they affect the percentiles of surplus.

3.3 Make six new worksheets with a Vasicek model instead of CIR.

4. A little theory

The lognormal distribution is useful for modelling asset returns and insurance claims. Assume that the random variable X has a lognormal distribution with parameters μ, σ^2. Prove the following:

1. The cumulative distribution of X is $F(x)=\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)$.
2. The probability density of X is $f(x)=F'(x)=\varphi\left(\frac{\ln(x)-\mu}{\sigma}\right)\frac{1}{\sigma x}$.
3. $\mathrm{E}(X)=\exp\left(\mu+\frac{1}{2}\sigma^2\right)$.
4. $\mathrm{Var}(X)=\mathrm{E}^2(X)\cdot\left(\exp(\sigma^2)-1\right)$.
5. For fixed K>0: $\mathrm{E}(X-K)^+ \quad = \quad \mathrm{E}(X)\cdot\Phi\left(\frac{\mu+\sigma^2-\ln(K)}{\sigma}\right)-K\cdot\Phi\left(\frac{\mu-\ln(K)}{\sigma}\right)$.
6. For fixed K>0: $\mathrm{E}(K-X)^+ \quad = \quad K\cdot\Phi\left(\frac{\ln(K)-\mu}{\sigma}\right)-\mathrm{E}(X)\cdot\Phi\left(\frac{\ln(K)-\mu-\sigma^2}{\sigma}\right)$.

7. Show that $\mathrm{VaR}_\alpha(F) = \exp\left(\mu + \sigma \cdot \Phi^{-1}(\alpha)\right)$

8. Show that $\mathrm{CTE}_\alpha(F) = \mathrm{E}\left(\mathrm{X} \mid \mathrm{X} > F^{-1}(\alpha)\right) = \frac{1}{1-\alpha} \cdot \exp\left(\mu + \frac{1}{2}\sigma^2\right) \cdot \Phi\left(\sigma + \Phi^{-1}(1-\alpha)\right)$.

9. Show numerically that $\frac{\mathrm{VaR}_{0.995}(F) - \mathrm{E}(X)}{\sqrt{\mathrm{Var}(X)}} \approx 3$ for $\sigma < 0.2$. This is the reason for Solvency II capital requirements being expressed as "3 standard deviations".

Here $\Phi(x)$ is the cumulative standard normal distribution and $\varphi(x) = \Phi'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.